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The Eccentric-Distance Sum of Cycles and Related Graphs

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Abstract. Let $G = (V, E)$ be a simple connected graph. The eccentric-distance sum of G is defined as $\xi^{ds}(G) = \sum\limits_{u \in V(G)} e(u)D(u)$ where $e(u)$ is the eccentricity of the vertex u in G and $D(u)$ is the sum of distances between u and all other vertices of G. In this paper, we establish formulae to calculate the eccentric-distance sum for some cycle related graphs, namely C_n, complement of C_n, shadow of C_n and the line graph of C_n. Also, it is shown that, the eccentric-distance sum of C_n is less than the eccentric-distance sum of shadow of C_n for all $n \geq 3$.

Key words: Distance, Eccentricity, Eccentric-Distance Sum.

Mathematics Subject classification 2010: 05C12.

1. Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by n and p respectively. For basic definitions and terminologies we refer to [1]. For vertices u and v in a connected graph G, the distance $d(u, v)$ is the length of a shortest $u - v$ path in G. A $u - v$ path of length $d(u, v)$ is called a $u - v$ *geodesic*. The *eccentricity* $e(v)$ of a vertex v in G is the maximum distance from v and a vertex of G. The minimum eccentricity among the vertices of G is the *radius*, *rad* G or $r(G)$ and the maximum eccentricity is its *diameter*, *diam* G of G. A $u - v$ walk of G is a finite, alternating sequence $u = u_0e_1u_1e_2 \cdots, e_nu_n = v$ of vertices and edges in G beginning with vertex u and ending with vertex v such that

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$e_i = u_{i-1}u_i$, $i = 1, 2, \cdots, n$. The number n is called the *length* of the walk. A walk in which all the vertices are distinct is called a *path*. A closed walk $u_0, u_1, u_2, \cdots, u_n$ in which $n \geq 3$ and $u_0, u_1, u_2, \cdots, u_{n-1}$ are distinct is called a *cycle* of length n and is denoted by C_n. The complement $\bar{G}$ of a simple graph G is a simple graph with vertex set V, two vertices being adjacent in $\bar{G}$ if and only if they are not adjacent in G. The line graph $L(G)$ is a graph in which the vertices are the lines of G and two points in $L(G)$ are adjacent if and only if the corresponding lines are adjacent in G. The *shadow graph* $S(G)$ of a connected graph G is constructed by taking two copies of G say $G^{'}$ and $G^{''}$. Join each vertex $u^{'}$ in $G^{'}$ to the neighbours of the corresponding vertex $u^{''}$ in $G^{''}$. The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G(V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. The sum $G_1 + G_2$ is the graph $G_1 \cup G_2$ together with all the lines joining points of V_1 to the points of V_2 . In [2], Gupta, Singh and Madan introduced a novel topological descriptor which is called eccentric-distance sum index (EDS) and then the concept was studied by various authors. The eccentric-distance sum of G is defined as $\xi^{ds}(G) = \sum\limits_{u \in V(G)} e(u)D(u)$ where $e(u)$ is the eccentricity of the vertex u in G and $D(u)$ is the sum of distances between u and all other vertices of G. In this paper, we establish formulae to calculate the eccentric-distance sum for some cycle related graphs, namely C_n, complement of C_n, Shadow of C_n and the line graph of C_n.
Throughout this paper G denotes a connected graph with at least three vertices.

Observation 1.1. *[2]* $\xi^{ds}(K_n) = n(n-1)$.

Observation 1.2. *$L(G)$ is isomorphic to G if and only if G is a cycle.*

2. Main results

Theorem 2.1. *The eccentric distance sum of, the sum of two cycles of length n is* $\xi^{ds}(C_n + C_n) = 2n \times \lfloor n/2 \rfloor \times [n + (\sum\limits_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)]$

Proof. Clearly the graph $C_n + C_n$ has $2n$ number of vertices.

$$e(v_i) = \lfloor n/2 \rfloor \text{ where } i = 1, 2, 3, \ldots, 2n$$

$$
\begin{aligned}
D(v_i) &= 1+1+2+\cdots+\lfloor (n-1)/2\rfloor+\lfloor n/2\rfloor+\underbrace{(1+1+\cdots+1)}_{(n\ \text{times})}\\
&= 0+0+1+1+2+\cdots+\lfloor (n-1)/2\rfloor+\lfloor n/2\rfloor+n(1)\\
&= [0+0+1+1+2+\cdots+\lfloor (n-1)/2\rfloor+\lfloor n/2\rfloor]+n\\
&= [\sum_{j=1}^{n+1}\lfloor (j-1)/2\rfloor]+n
\end{aligned}
$$

$$
\begin{aligned}
\xi^{ds}(C_n+C_n) &= \sum_{i=1}^{2n} e(v_i)D(v_i)\\
&= e(v_1)D(v_1)+\cdots+e(v_{2n})D(v_{2n})
\end{aligned}
$$

$$= \lfloor n/2\rfloor\,[(\sum_{j=1}^{n+1}\lfloor (j-1)/2\rfloor)+n]+\cdots+\lfloor n/2\rfloor\,[(\sum_{j=1}^{n+1}\lfloor (j-1)/2\rfloor)+n](2n\ \text{times})$$

$$= 2n\,\lfloor n/2\rfloor\,[(\sum_{j=1}^{n+1}\lfloor (j-1)/2)\rfloor)+n]$$

Hence $\xi^{ds}(C_n+C_n) = 2n\times\lfloor n/2\rfloor\times[n+(\sum_{i=1}^{n+1}\lfloor (i-1)/2)\rfloor)]$ □

Remark 2.2. $\xi^{ds}(C_n) = n\times\lfloor n/2\rfloor\times(\sum_{i=1}^{n+1}\lfloor (i-1)/2\rfloor)$

Proof. The eccentricity of any vertex in (C_n+C_n) is same as the eccentricity of any vertex in C_n. Also, the distance sum of any vertex in (C_n+C_n) is equal to n plus the distance sum of any vertex in C_n. Thus $\xi^{ds}(C_n) = n\times\lfloor n/2\rfloor\times(\sum_{i=1}^{n+1}\lfloor (i-1)/2\rfloor)$. □

Theorem 2.3. *The eccentric distance sum of the sum of two cycles of length n and m where $n\neq m$ is* $\xi^{ds}(C_n+C_m) = n\times\lfloor n/2\rfloor\times[m+(\sum_{i=1}^{n+1}\lfloor (i-1)/2\rfloor)]+$ $m\times\lfloor m/2\rfloor\times[n+(\sum_{i=1}^{m+1}\lfloor (i-1)/2\rfloor)]$

Proof. Consider the graph C_n+C_m where $n\neq m$

Clearly it contains $n+m$ number of vertices.

$$e(v_i) = \lfloor n/2\rfloor \text{for all } i=1,2,3,\ldots,n$$

$$e(v_i) = \lfloor m/2\rfloor \text{ for all } i=n+1,\ldots,m$$

$$D(v_i) = 1+1+2+\cdots+\lfloor (n-1)/2\rfloor+\lfloor n/2\rfloor+\underbrace{(1+1+\cdots+1)}_{(m\ \text{times})}$$

for all $i=1,2,3,\cdots,n$

$$= 0 + 0 + 1 + 1 + 2 + \cdots + \lfloor (n-1)/2 \rfloor + \lfloor n/2 \rfloor + \underbrace{(1 + 1 + \cdots + 1)}_{(m \text{ times})}$$

$$= [\sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor] + m \quad \text{for all } i = 1, 2, 3, \ldots, n$$

$$D(v_i) = 1 + 1 + 2 + \cdots + \lfloor (m-1)/2 \rfloor + \lfloor m/2 \rfloor + \underbrace{(1 + 1 + \cdots + 1)}_{(n \text{ times})}$$

$$\text{for all } i = n+1, \ldots, m$$

$$= 0 + 0 + 1 + 1 + 2 + \cdots + \lfloor (m-1)/2 \rfloor + \lfloor m/2 \rfloor + \underbrace{(1 + 1 + \cdots + 1)}_{(n \text{ times})}$$

$$= [\sum_{j=1}^{m+1} \lfloor (j-1)/2 \rfloor] + n \; for \; all \; i = n+1, \ldots, m$$

$$\xi^{ds}(C_n + C_m) = \sum_{i=1}^{n+m} e(v_i) D(v_i)$$

$$= e(v_1)D(v_1) + \cdots + e(v_n)D(v_n) + e(v_{n+1})D(v_{n+1}) + \cdots + e(v_m)D(v_m)$$

$$= \lfloor n/2 \rfloor [(\sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor) + m] + \cdots + \lfloor n/2 \rfloor [(\sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor) + m]$$

$$+ \lfloor m/2 \rfloor [(\sum_{j=1}^{m+1} \lfloor (j-1)/2 \rfloor) + n] + \cdots + \lfloor m/2 \rfloor [(\sum_{j=1}^{m+1} \lfloor (j-1)/2 \rfloor) + n]$$

$$= n \times \lfloor n/2 \rfloor \times [m + \sum_{j=1}^{n+1} \lfloor (j-1)/2 \rfloor] + m \times \lfloor m/2 \rfloor \times [n + (\sum_{i=1}^{m+1} \lfloor (i-1)/2 \rfloor)]$$

Hence

$$\xi^{ds}(C_n + C_m) = n \times \lfloor n/2 \rfloor [m + (\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)] + m \times \lfloor m/2 \rfloor [n + (\sum_{i=1}^{m+1} \lfloor (i-1)/2 \rfloor)].$$

□

Remark 2.4. $\xi^{ds}(C_n + C_m) \neq \xi^{ds}(C_{n+m})$.

Proof. By remark 2.2,$\xi^{ds}(C_{n+m}) = (n+m) \times \lfloor (n+m)/2 \rfloor \times (\sum_{i=1}^{n+m+1} \lfloor (i-1)/2 \rfloor)$

By theorem 2.3,$\xi^{ds}(C_n + C_m) = n \times \lfloor n/2 \rfloor \times [m + (\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)] + m \times \lfloor m/2 \rfloor$

$$\times [n + (\sum_{i=1}^{m+1} \lfloor (i-1)/2 \rfloor)]$$

Hence the result follows. □

Theorem 2.5. *For $n \geq 5, \xi^{ds}(\overline{C_n}) = 2n(n+1)$.*

Proof. $e(v_i) = 2$ for all $i = 1, 2, \cdots, n$

$$\begin{aligned} D(v_i) &= n+1 \text{ for all } i = 1,2,\cdots,n \\ \xi^{ds}(\overline{C_n}) &= \sum_{i=1}^{n} e(v_i)D(v_i) \\ &= e(v_1)D(v_1) + \cdots + e(v_n)D(v_n) \\ &= 2(n+1) + \cdots + 2(n+1)(n \text{ times}) \\ &= n \times 2 \times (n+1) = 2n(n+1) \end{aligned}$$

□

Remark 2.6. *For $n = 3, 4, (\overline{C_n})$ is a disconnected graph and so eccentric distance sum cannot be determined.*

Remark 2.7. *Eccentric distance sum cannot be determined for $(\overline{C_n + C_n})$.*

Proof. $(\overline{C_n + C_n})$ is the union of $(\overline{C_n})$ and $(\overline{C_n})$.
That is $(\overline{C_n + C_n}) = (\overline{C_n}) \cup (\overline{C_n})$
$(\overline{C_n}) \cup (\overline{C_n})$ is a disconnected graph.
Thus the result follows. □

Theorem 2.8. *For $n \geq 6$, $\xi^{ds}(\overline{C_n}) < \xi^{ds}(C_n)$.*

Proof. For $n \geq 6$, $n+1 < \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor$

$$\begin{aligned} &\Rightarrow \; n(n+1) < n \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor \\ &\Rightarrow 2n(n+1) < 2n \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor \\ &\qquad\qquad < n \lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor \\ &\Rightarrow 2n(n+1) < n \lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor \end{aligned}$$

Thus $\xi^{ds}(\overline{C_n}) < \xi^{ds}(C_n)$ for $n \geq 6$. □

Theorem 2.9. *If two graphs are isomorphic then their eccentric distance sum is equal.*

Proof. Let $G_1$and G_2be two graphs which are isomorphic. Then the eccentricity of every vertex in $G_1$and G_2 will be equal and the distance sum of every vertex in $G_1$and G_2will be equal. Hence the eccentric distance sum of the two graphs will be equal. □

Result 2.10. $\xi^{ds}(C_n + C_n) = \xi^{ds}(K_{2n})$ *for* $n = 3$.

Proof. The graph $C_3 + C_3$ is isomorphic to the complete graph with six vertices K_6.Thus $\xi^{ds}(C_3 + C_3) = \xi^{ds}(K_6)$.

We can prove the same result by giving particular value for $n = 3$
We know that $\xi^{ds}(C_n + C_n) = 2n \times \lfloor n/2 \rfloor \times [n + (\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)]$

$\xi^{ds}(C_3 + C_3) = 2 \times 3 \times \lfloor 3/2 \rfloor \, [3 + 0 + 0 + 1 + 1] = 30$

We know that $\xi^{ds}(K_n) = n(n-1)$

$$\xi^{ds}(K_6) = 6(6-1) = 30$$
$$\xi^{ds}(C_3 + C_3) = \xi^{ds}(K_6)$$
□

Result 2.11. *For* $n = 5$,$\xi^{ds}(C_n) = \xi^{ds}(\overline{C_n})$.

Proof. The cycle graph on 5 vertices, C_5 is the unique self- complementary graph (up to graph isomorphism)
That is C_5 is isomorphic to its complement.
Thus $\xi^{ds}(C_5) = \xi^{ds}(\overline{C_5})$ Also, We can show the same result by giving particular value for $n = 5$ in the formula

$$\xi^{ds}(C_n) = n \times \lfloor n/2 \rfloor \times [\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]$$

$$\xi^{ds}(C_5) = 5 \times \lfloor 5/2 \rfloor \times [\sum_{i=1}^{6} \lfloor (i-1)/2 \rfloor]$$
$$= 5 \times 2 \times [0 + 0 + 1 + 1 + 2 + 2] = 60$$
$$\xi^{ds}(\overline{C_n}) = 2n(n+1) = 60$$
$$\xi^{ds}(C_5) = \xi^{ds}(\overline{C_5}).$$
□

Theorem 2.12. $\xi^{ds}(C_n) = \xi^{ds}(L(C_n))$.

Proof. By observation 1.2, C_n is isomorphic to $L(C_n)$. Thus $\xi^{ds}(C_n) = \xi^{ds}(L(C_n))$. □

Theorem 2.13. *For* $n = 3$, $\xi^{ds}(S(C_n)) = 4n\lceil n/2\rceil [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]$.

Proof. $e(v_i) = \lceil n/2\rceil$ for all $i = 1, 2, 3, \cdots, n$

$$e(v_i') = \lceil n/2\rceil \text{ for all } i = 1, 2, \cdots, n$$

$$\begin{aligned} D(v_i) &= [2\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] + 2 \text{ for all } i = 1, 2, 3, \cdots, n \\ &= 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] \end{aligned}$$

$$\begin{aligned} D(v_i') &= [2\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] + 2 \text{ for all } i = 1, 2, 3, \cdots, n \\ &= 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] \end{aligned}$$

For $n = 3$

$$\begin{aligned} \xi^{ds}(S(C_n)) &= \sum_{u \in V(S(C_n))} e(u)D(u) \\ &= e(v_1)D(v_1) + \cdots + e(v_n)D(v_n) + e(v_1')D(v_1') + \cdots + e(v_n')D(v_n') \\ &= \end{aligned}$$

$$\lceil n/2\rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] + \cdots + \lceil n/2\rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)$$

$$+ 1] + \lceil n/2\rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] + \cdots + \lceil n/2\rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1]$$

$$\begin{aligned} &= 2n[\lceil n/2\rceil \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1]] \\ &= 4n[\lceil n/2\rceil [(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1]] \end{aligned}$$

Hence $\xi^{ds}(S(C_n)) = 4n\lceil n/2\rceil [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]$. □

Theorem 2.14. *For* $n \geq 4$, $\xi^{ds}(S(C_n)) = 4n \times \lfloor n/2 \rfloor \times [1 + (\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)]$.

Proof. Clearly $S(C_n)$ has $2n$ number of vertices

$$e(v_i) = \lfloor n/2 \rfloor \text{ for all } i = 1, 2, 3, \cdots, n$$

$$
\begin{aligned}
e(v_i^{'}) &= \lfloor n/2 \rfloor \text{ for all } i = 1, 2, \cdots, n \\
D(v_i) &= [2(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)] + 2 \text{ for all } i = 1, 2, 3, \cdots, n \\
&= 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] \\
D(v_i^{'}) &= [2(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)] + 2 \text{ for all } i = 1, 2, 3, \cdots, n \\
&= 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] \\
\xi^{ds}(S(C_n)) &= \sum_{u \in V(S(C_n))} e(u) D(u) \\
&= e(v_1)D(v_1) + \cdots + e(v_n)D(v_n) + e(v_1^{'})D(v_1^{'}) + \cdots + e(v_n^{'})D(v_n^{'}) \\
&= \lfloor n/2 \rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] + \cdots + \lfloor n/2 \rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)
\end{aligned}
$$

$$
+ 1] + \lfloor n/2 \rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] + \cdots + \lfloor n/2 \rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1]
$$

$$
\begin{aligned}
&= 2n[\lfloor n/2 \rfloor \times 2[(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1]] \\
&= 4n \lfloor n/2 \rfloor [(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1]
\end{aligned}
$$

Hence $\xi^{ds}(S(C_n)) = 4n \times \lfloor n/2 \rfloor \times [1 + (\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor)]$. □

Theorem 2.15. *$\xi^{ds}(C_n) < \xi^{ds}(S(C_n))$ for $n \geq 3$.*

Proof. First we prove for $n \geq 4$.

$$
\begin{aligned}
\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor &< 1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor \\
\lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor &< \lfloor n/2 \rfloor [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] \\
n \lfloor n/2 \rfloor \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor &< n \lfloor n/2 \rfloor [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor] \\
&< 4n \lfloor n/2 \rfloor [1 + \sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor]
\end{aligned}
$$

$$i.e.n\left\lfloor n/2\right\rfloor\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor < 4n\left\lfloor n/2\right\rfloor[1+\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor]$$

$$\Rightarrow \xi^{ds}(C_n) < \xi^{ds}(S(C_n)) for n \geq 4$$

For $n = 3$

$$\begin{aligned}
\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor &< 1+\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor \\
\left\lfloor n/2\right\rfloor\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor &< \ \left\lfloor n/2\right\rfloor[1+\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor] \\
&\leq \ \left\lceil n/2\right\rceil[1+\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor] \quad (\text{since} \quad \left\lfloor n/2\right\rfloor \leq \left\lceil n/2\right\rceil) \\
\left\lfloor n/2\right\rfloor\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor &< \ \left\lceil n/2\right\rceil[1+\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor] \\
n\left\lfloor n/2\right\rfloor\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor &< n\left\lceil n/2\right\rceil[1+\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor] \\
&< 4n\left\lceil n/2\right\rceil[1+\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor] \\
n\left\lfloor n/2\right\rfloor\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor &< 4n\left\lceil n/2\right\rceil[1+\sum_{i=1}^{n+1}\left\lfloor (i-1)/2\right\rfloor]
\end{aligned}$$

Thus $\xi^{ds}(C_n) < \xi^{ds}(S(C_n))$ for $n \geq 3$. □

Theorem 2.16. $\xi^{ds}(S(\overline{C_n})) = 8n(n+2)$ *for* $n \geq 5$.

Proof. $S(\overline{C_n})$ has $2n$ vertices

$$\begin{aligned}
e(v_i) &= 2 \ \text{ for all } \ i = 1,2,3,\cdots,n \\
e(v_i^{'}) &= 2 \ \text{ for all } \ i = 1,2,\cdots,n \\
D(v_i) &= 2(n+2) \ \text{ for all } \ i = 1,2,3,\cdots,n \\
D(v_i^{'}) &= 2(n+2) \ \text{ for all } \ i = 1,2,3,\cdots,n
\end{aligned}$$

$$
\begin{aligned}
\xi^{ds}(S(\overline{C_n})) &= \sum_{u \in V(S(\overline{C_n}))} e(u)D(u) \\
&= e(v_1)D(v_1) + \cdots + e(v_n)D(v_n) + e(v_1^{'})D(v_1^{'}) + \cdots + e(v_n^{'})D(v_n^{'}) \\
&= 2[2(n+2)] + \cdots + 2[2(n+2)] + 2[2(n+2)] + \cdots + 2[2(n+2)] \\
&= 2n[2 \times (2(n+2))] \\
&= 8n(n+2)
\end{aligned}
$$

□

Result 2.17. *$\xi^{ds}(S(\overline{C_n})) = \xi^{ds}(S(C_n))$ for $n = 5$.*

Proof. Since $C_n$is isomorphic to its complement , the result follows.

Aliter:

$$
\begin{aligned}
\xi^{ds}(S(\overline{C_n})) &= 8n(n+2) \\
\xi^{ds}(S(\overline{C_5})) &= 8 \times 5(5+2) \\
&= 280 \qquad (1) \\
\xi^{ds}(S(C_n)) &= 4n \lfloor n/2 \rfloor [(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] \\
\xi^{ds}(S(C_5)) &= 4 \times 5 \times \lfloor 5/2 \rfloor [(\sum_{i=1}^{6} \lfloor (i-1)/2 \rfloor) + 1] \\
&= 4 \times 5 \times 2[0+0+1+1+2+2+1] \\
&= 280 \qquad (2)
\end{aligned}
$$

From (1) and (2)

$\xi^{ds}(S(\overline{C_n})) = \xi^{ds}(S(C_n))$ for $n = 5$ □

Result 2.18. *$\xi^{ds}(S(\overline{C_n})) < \xi^{ds}(S(C_n))$ for $n \geq 6$*

Proof. We find the values of $\xi^{ds}(S(\overline{C_n}))$ and$\xi^{ds}(S(C_n))$ as follows:

When $n = 6$, $\xi^{ds}(S(\overline{C_n})) = 8n(n+2) = 384$

$$\xi^{ds}(S(C_n)) = 4n \lfloor n/2 \rfloor [(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] = 720$$

When $n = 7$, $\xi^{ds}(S(\overline{C_n})) = 8n(n+2) = 504$

$$\xi^{ds}(S(C_n)) = 4n \lfloor n/2 \rfloor [(\sum_{i=1}^{n+1} \lfloor (i-1)/2 \rfloor) + 1] = 1092$$

When $n = 8$, $\xi^{ds}(S(\overline{C_n})) = 8n(n+2) = 640$

$$\xi^{ds}(S(C_n)) = 4n\lfloor n/2\rfloor[(\sum_{i=1}^{n+1}\lfloor(i-1)/2\rfloor) + 1] = 2176$$

When $n = 9$, $\xi^{ds}(S(\overline{C_n})) = 8n(n+2) = 792$

$$\xi^{ds}(S(C_n)) = 4n\lfloor n/2\rfloor[(\sum_{i=1}^{n+1}\lfloor(i-1)/2\rfloor) + 1] = 3024$$

Thus we see that $\xi^{ds}(S(\overline{C_n})) < \xi^{ds}(S(C_n))$ for $n \geq 6$ □

3. Conclusion

In this paper we have found the eccentric distance sum of, the sum of two cycles of length n, the eccentric distance sum of a cycle , the eccentric distance sum of complement of a cycle , the eccentric distance sum of the line graph of a cycle , the eccentric distance sum of the shadow graph of a cycle and we conclude that the eccentric distance sum of the complement of a cycle is less than the eccentric distance sum of a cycle for $n \geq 6$, the eccentric distance sum of a cycle is less than the eccentric distance sum of the shadow of a cycle for $n \geq 3$ and the eccentric distance sum of the shadow of complement of a cycle is less than the eccentric distance sum of the shadow of a cycle for $n \geq 6$.

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