J. Comp.Matha. Vol.2(1), (2018). ISSN: 2456-8686

Original Research Article

SACRED HEART RESEARCH PUBLICATIONS

Journal of Computational Mathematica

Journal homepage: www.shcpub.edu.in

COMPUTATIONAL MATHEMATICA

ISSN: 2456-8686

Skolem Mean Labeling of Four Star Graphs $K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $|b-(a_1+a_2+a_3)| = 4$

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Received on 21 January 2018, Accepted on 15 May 2018

Abstract. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $|b-(a_1+a_2+a_3)| = 4$.

Key words: Skolem mean graph, skolem mean labeling, star graphs.

1. Introduction

In this paper all graphs are finite, simple and undirected. Terms and notations are used in the sense of Harary [3]. Much work is done by many researchers on skolem mean labelling [1], [2] and [3]. In [5], [6] and [7] some results are proved in four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ on skolem mean labelling. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $|b-(a_1+a_2+a_3)| = 4$. That is when $b = (a_1+a_2+a_3)+4$ and $b = (a_1+a_2+a_3)-4$.

Definition 1.1. A graph $G = (V, E)$ with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to $1, 2, \cdots, p$ such that the induced map f^* from the edge set of G to $2, 3, \cdots, p$ defined by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if} \quad f(u)+f(v) \quad \text{is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if} \quad f(u)+f(v) \quad \text{is odd} \end{cases}$$

the resulting edges get unique labels from the set $2, 3, \cdots, p$.

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Theorem 1.2. *The four star $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \leq a_2 \leq a_3$ is a skolem mean graph if $|b - (a_1 + a_2 + a_3)| = 4$.*

Proof. Let $A_i = \sum_{k=1}^{i} a_k$. That is, $A_1 = a_1$; $A_2 = a_1 + a_2$ and $A_3 = a_1 + a_2 + a_3$. Consider the graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$. Let $V = \bigcup_{k=1}^{4} V_k$ be the vertex set of G where $V_k = \{v_{k,i} : 0 \leq i \leq a_k\}$ for $1 \leq k \leq 3$ and $V_4 = \{v_{4,i} : 0 \leq i \leq b\}$. Let $E = \bigcup_{k=1}^{4} E_k$ be the edge set of G where $E_k = \{v_{k,0}v_{k,i} : 0 \leq i \leq a_k\}$ for $1 \leq k \leq 3$ and $E_4 = \{v_{4,0}v_{4,i} : 0 \leq i \leq b\}$.

The condition $|b - (a_1 + a_2 + a_3)| = 4 \Rightarrow b = A_3 - 4 \, or \, b = A_3 + 4$.

That is, there are two cases viz. $b = A_3 - 4$ and $b = A_3 + 4$.

Let us prove in each of the two cases the graph G is a skolem mean graph.

Case 1: Let $b = A_3 + 4$

G has $A_3 + b + 4 = 2A_3 + 8$ vertices and $A_4 + b = 2A_3 + 4$ edges.

The vertex labeling

$f : V \to \{1, 2, \cdots, A_3 + b + 4 = 2A_3 + 8\}$ is defined as follows:

$$f(v_{1,0}) = 1; \quad f(v_{2,0}) = 2; \quad f(v_{3,0}) = 4;$$

$$f(v_{4,0}) = A_3 + b + 3 = 2A_3 + 7$$

$$f(v_{1,i}) = 2i + 4 \qquad 1 \leq i \leq a_1$$

$$f(v_{2,i}) = 2A_1 + 2i + 4 \qquad 1 \leq i \leq a_2$$

$$f(v_{3,i}) = 2A_2 + 2i + 4 \qquad 1 \leq i \leq a_3$$

$$f(v_{4,i}) = 2i + 11 \leq i \leq b - 2 = A_3 + 2$$

$$f(v_{4,b-1}) = A_3 + b + 2 = 2A_3 + 6$$

$$f(v_{4,b}) = A_3 + b + 4 = 2A_3 + 8$$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $3+i$ for $1 \leq i \leq a_1$ (edge labels are $4, 5, \cdots, a_1 + 3 = A_1 + 3$), $v_{2,0}v_{2,i}$ is A_1+3+i for $1 \leq i \leq a_2$ (edge labels are $A_1+4, A_1+5, \cdots, A_2+3$), $v_{3,0}v_{3,i}$ is A_2+4+i for $1 \leq i \leq a_3$ (edge labels are $A_2+5, A_2+6, \cdots, A_3+4$), $v_{4,0}v_{4,i}$ is A_3+4+i for $1 \leq i \leq b-2 = A_3+2$ (edge labels are $A_3+5, A_3+6, \cdots, 2A_3+6$), $v_{4,0}v_{b-1}$ is $2A_3 + 7$ and $v_{4,0}v_{4,b}$ is $2A_3 + 8$.

These induced edge labels of graph G are unique.

Hence G is a skolem mean graph.

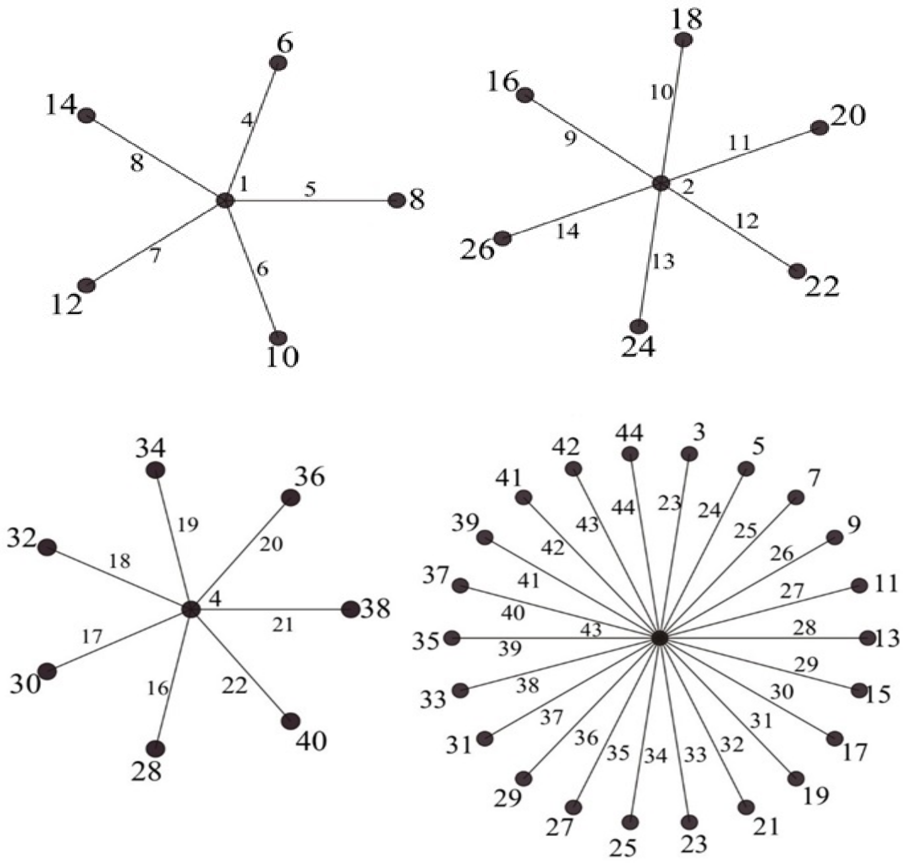

FIGURE 1. $K_{1,5} \cup K_{1,6} \cup K_{1,7} \cup K_{1,22}$

Case 2: Let $b = A_3 - 4$

G has $A_3 + b + 4 = 2A_3$ vertices and $A_3 + b = 2A_3 - 4$ edges.

The vertex labeling $f : V \to \{1, 2, \cdots, A_3 + b + 4 = 2A_3\}$ is defined as follows:

$$f(v_{1,0}) = 2; \quad f(v_{2,0}) = 4; \quad f(v_{3,0}) = 6;$$

$$f(v_{4,0}) = A_3 + b + 4 = 2A_3$$

$$f(v_{1,i}) = 2i - 1 \qquad 1 \leq i \leq a_1$$

$$f(v_{2,i}) = 2A_1 + 2i - 1 \qquad 1 \leq i \leq a_2$$

$$f(v_{3,i}) = 2A_2 + 2i - 1 \qquad 1 \leq i \leq a_3$$

$$f(v_{4,i}) = 2i + 6 \qquad 1 \leq i \leq b$$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $1+i$ for $1 \leq i \leq a_1$ (edge labels are $2, 3, \cdots, a_1+1 = A_1+1$), $v_{2,0}v_{2,i}$ is A_1+2+i for $1 \leq i \leq a_2$ (edge labels are $A_1+3, A_1+4, \cdots, A_2+2$), $v_{3,0}v_{3,i}$ is A_2+3+i for $1 \leq i \leq a_3$ (edge labels are $A_2+4, A_2+5, ..., A_2+a_3+3 = A_3+3$), $v_{4,0}v_{4,i}$ is A_3+3+i for $1 \leq i \leq b = A_3-4$ (edge labels are $A_3+4,\ A_3+5, \cdots, A_3+3+b = A_3+3+A_3-4 = 2A_3-1$).

These induced edge labels of graph G are unique.

Hence G is a skolem mean graph. $\square$

Example 1.3.

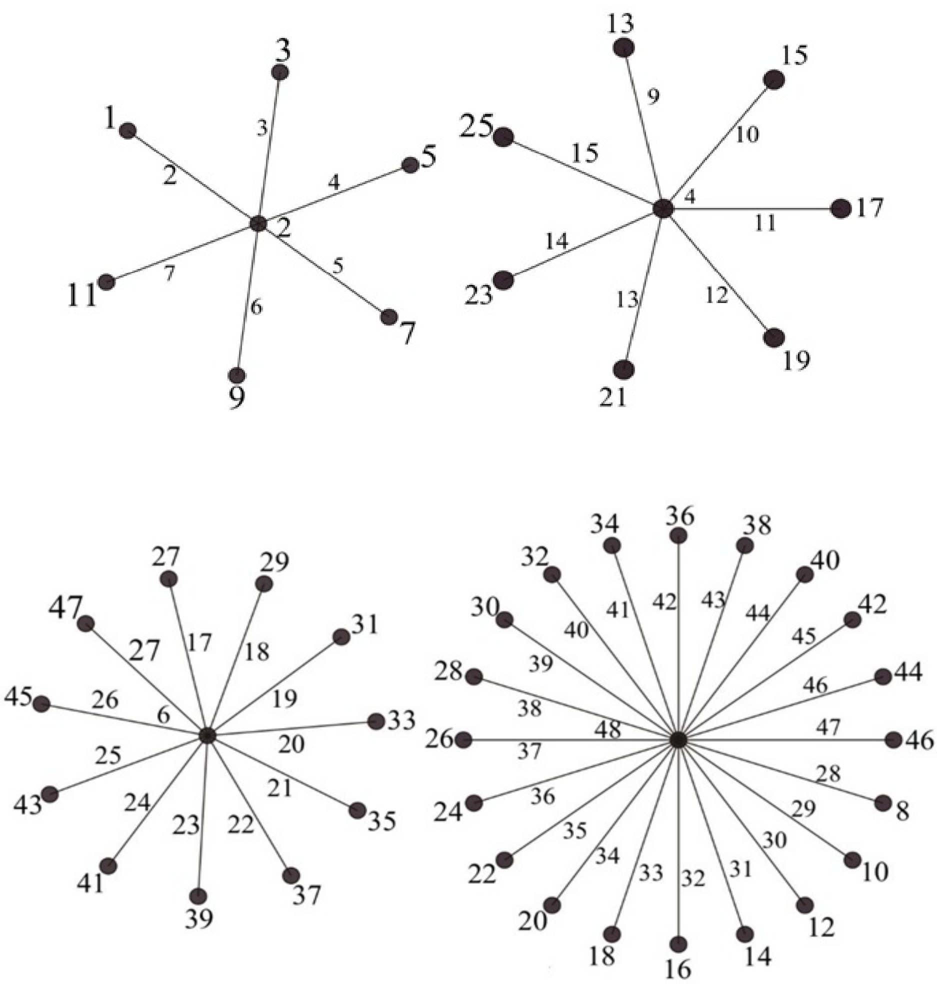

FIGURE 2. $K_{1,6} \cup K_{1,7} \cup K_{1,1} \cup K_{1,20}$

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